



Comparison of Model and Experimental Results of Elastoplastic Structure Loaded with Bending Moment and Torsion

Purwanto^{1*}, Han Ay Lie¹, and Blinka Hernawan Prasetya¹

¹*Department of Civil Engineering, Universitas Diponegoro, Jl. Prof. Soedarto, SH., Semarang, 50275, Indonesia*

* *Corresponding author: purwatrend@gmail.com*

(Received: 11th September 2023 ; Revised: 20th October 2023 ; Accepted: 6th November 2023)

Abstract: Experimental testing and numerical analysis were conducted on steel structure with 20×20 mm rectangular section loaded with combination of bending moment (M) and torsion (T), used to validated the previous study. Specimen used in the research was cantilever grid beam with fixed constraint at the end of the beam. The specimen was incrementally loaded parallel with gravity direction until failure occurred. Load, displacement and strain at observed point were recorded to analyze failure criteria based on Von Mises criteria. Numerical model using DIANA was used to analyze further the internal forces and stress distribution at the beam structure. 3D solid model was used for the numerical analysis in order to represent the experimental results. Experimental and numerical analysis results were in good agreement compared to the previous study. The experimental strain data shows that plastic joints formed gradually in one location and followed in other locations. Analysis using Von Mises criteria shows that the yield of the structure was significantly affected by bending with proportion of $\pm 90\%$. Numerical model located the same position of plastic joints compared to experimental. The stress distribution shows that the maximum stress is located at the vertical outer side of section which indicated that the bending stress is dominant.

Keywords: Cantilever; DIANA; yield criteria; bending moment; torsion

1. Introduction

Structures loaded with combination of bending moment (M) and torsion (T) are often found in the field, this type of structures can be found on the grid construction or other type of construction. Plastification of the structural element is usually designed based on the magnitude of bending moment (M). Structures loaded with high torsion (T), the plastification is based on the interaction of the bending (M) and torsion (T). Analytical study of elastoplastic structural element impacted by the combination of bending and torsion has been done by [1], [2]. Research about the structural behavior of a beam loaded by combination of torsion (T) and bending (M) also has been done by [3], [4], but most of the structures studied were hollow or I-section [5], [6] meanwhile a solid section was rarely studied. Experimental test and numerical modelling were carried to verify the plastification process in a structure impacted by the combination of bending moment (M) and torsion (T). One of the results from the experimental and numerical was the ultimate capacity of the beam which then compared to the space frame limit design [7].

2. Literature review

Structure which gradually loaded with proportional increment will result in the increased number of the plastic hinge. The increasing of plastic hinge will decrease the structural strength and may lead to structural failure. Plastic hinge can be observed by incrementally load the structure up to failure so that in the process, plastic hinge will occur at the critical location. Steel material has an isotropic behavior, independent from the hydrostatic stress and has an equal strength in compression and tension. Von Mises criteria will be used for the yield criteria [8]. Von Mises criteria state the yield of the material is determined by the octahedral shear stress or strain distortion energy working at the material. In terms of principal stress, Von Mises criteria can be written as:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_0^2 \quad (1)$$

In (σ, τ) plane, Von Mises can be written as [4]:

$$\sigma^2 + 3\tau^2 = \sigma_0^2 \quad (2)$$

For the steel structure loaded with combination of bending and torsion stress so the Von Mises criteria satisfies:

$$\begin{aligned} \sigma^2 + 3\tau^2 &= \sigma_y^2 = 3\tau_y^2 \\ \left(\frac{\sigma}{\sigma_y}\right)^2 + \left(\frac{3\tau}{3\tau_y}\right)^2 &= 1 \\ \left(\frac{\sigma}{\sigma_y}\right)^2 + \left(\frac{\tau}{\tau_y}\right)^2 &= 1 \end{aligned} \quad (3)$$

Expressed in terms of internal forces, equation can be written as:

$$\left(\frac{M}{M_y}\right)^2 + \left(\frac{T}{T_y}\right)^2 = 1 \quad (4)$$

with $\sigma_n = (\sigma_1, \sigma_2, \sigma_3)$ = principal stress, $\sigma_0 = \sigma_y$ = yield stress, σ = actual normal stress, τ = actual shear stress, M = actual bending moment, M_y = bending capacity of the beam at yield condition, T = actual torsion and T_y = torsion capacity of the beam at yield condition. The value of M_y dan T_y can be computed with:

$$M_y = \frac{1}{6} \times b \times h^2 \times \sigma_y \quad (5)$$

$$T_y = C_s \times b^2 \times h \times \tau_y \quad (6)$$

$$\tau_y = \frac{\sigma_y}{\sqrt{3}}$$

with b = width of the section, h = height of the section and C_s = constant (0.20875). Unit used in the Eq. (6) must be in kilogram (kg) centimeter (cm).

3. Research Method

3.1. Specimen and experimental setup

Experimental test of a beam to analyze the bending-torsion yielding in a space frame structure was conducted by using similar specimen from [1], [2]. Specimen used in this research was a 3D statically indetermined beam structure with 20×20 mm solid rectangular section, 3 specimens were prepared for the experiment. Fixed constraint was used at the end of each beam. Observed location for the yield is determined at point A, B and C which A and C are the fixed constraint points meanwhile B is located below the loading point. Steel used for the experiment has 390 MPa of yield (σ_y) strength and 200000 MPa of elastic moduli (E_s). Setup of the instruments, LVDTs, data logger, load cell and strain gauges, is based on [9]. Detail of the setup and specimen's dimension is shown at Fig. 1.

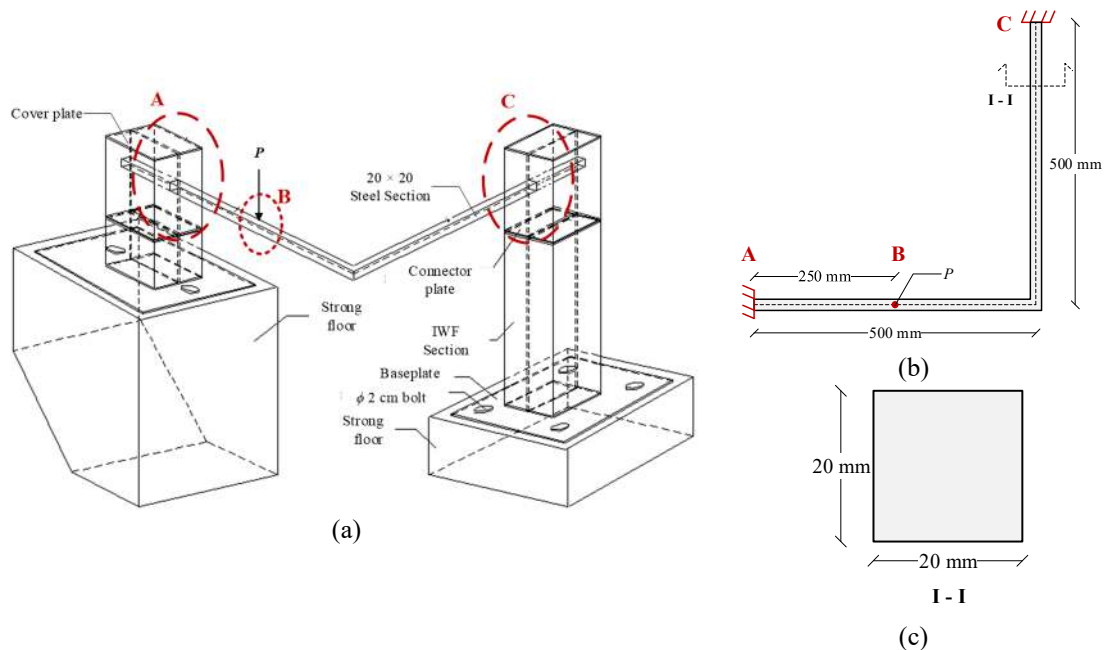


Fig. 1. (a) Test setup, (b) plan view and (c) section view of the beam specimen [10]

Incremental load was applied at the point B until failure occurred. Load cell was used to record the load data during the testing. LVDTs were installed at B and D (beam-column joint) to measure the displacement. Strain gauges were installed at A, B and C to measure strain which would be used to determine the yield at the observed points.

3.2. Numerical model

Numerical modelling was done using DIANA FEA 10.3 with 3D analysis (solid model) to realistically model the 3D statically indetermined beam. Element used for the analysis was a cube element with 8 main nodes and 12 additional nodes (CHX60, quadratic). Mesh size used for the model was chosen by 5 mm. Deformation control was used for the loading type with loading rate of 0.05 mm/step which also similar with the experimental. The model of the beam can be seen at Fig. 2. Output from the model is load (P), displacement (δ), internal forces and stress distribution.

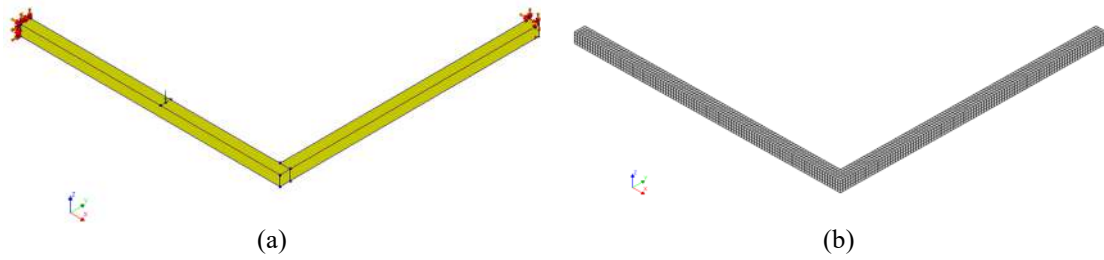


Fig. 2. (a) Model of the beam and (b) beam's meshing.

3.3. Analysis

Data recorded from the experiment were load (P), displacement (δ) and strain (ϵ). Load-displacement data (P - δ) recorded from experiment then used as the reference to build the numerical model. Model output was compared by load-displacement (P - δ) from experimental data to validate the model. Strain reading from strain gauge (ϵ) was used to determine the yield condition. Numerical model then was used to obtain the value of bending moment (M) and torsion (T) which used to evaluate the 3D statically indetermined structure plastification process based on Von Mises criteria.

4. Discussion

4.1. Results

Load-displacement (P - δ) responses from experimental result and DIANA FEA 10.3's numerical model is shown at Fig. 3. Results from [1], [2] and calculated capacity based on [7] analysis are also shown to compare and validate the current research's result.

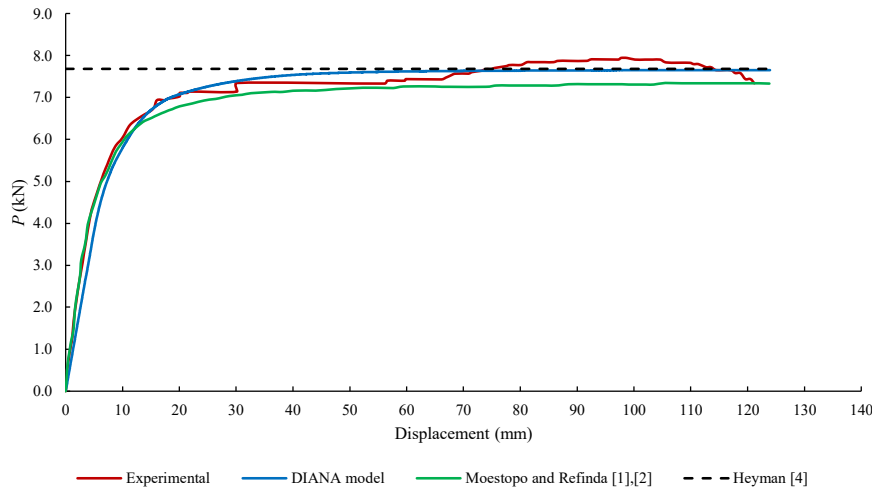


Fig. 3. Load-displacement graph of specimens.

Fig. 3 shows a non-linear response from the 3D structure which indicated that plastic joint was formed in the structure. Based on the strain data (ϵ), first plastic joint's location was located at point A (fixed constraint location), followed by point C (the other fixed constraint location) and ultimately occurred at point B (below the loading point P). Load-displacement (P - δ) shows similar results between experimental and numerical results. Recapitulation of ultimate load is presented at Table 1 and load when plastic joint was formed is presented at Table 2. Results show that numerical model from DIANA FEA 10.3 is accepted as experimental representative due to very similar output.

Comparison of Model and Experimental Results of Elastoplastic Structure Loaded with Bending Moment and Torsion

Table 1. Recapitulation of beam's ultimate load

No.	Sample	Ultimate load (kN)			
		Experimental	DIANA Model	Moestopo and Refinda [1], [2]	Heyman [7]
1	Sample 1	7.8388	7.654	7.34	7.68
2	Sample 2	7.8965	7.654	7.34	7.68
3	Sample 3	7.9538	7.654	7.34	7.68

Table 2. Recapitulation of beam's yield load

No.	Sample	Plastic joint location	Yield load (kN)		
			Experimental	DIANA Model	Moestopo and Refinda [1], [2]
1	Sample 1	A	3.605	3.487	3.27
		C	6.180	5.746	5.53
		B	6.924	6.875	6.29
2	Sample 2	A	3.950	3.487	3.27
		C	6.070	5.746	5.53
		B	6.980	6.875	6.29
3	Sample 3	A	3.780	3.487	3.27
		C	6.580	5.746	5.53
		B	6.690	6.875	6.29

4.2. Data analysis

Plastic joint was predictable by using Von Mises criteria, bending moment (M) and torsion (T) values were taken from DIANA FEA 10.3 to analyze the Von Mises criteria. Internal forces taken then used to predict and compare the result with the experiment. Bending moment (M) and torsion (T) values are shown at Table 3.

Table 3. Recapitulation of bending moment (M) and torsion (T)

Load (kN)	M (kNm)			T (kNm)		
	Point A	Point C	Point B	Point A	Point C	Point B
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.72	-0.01133	-0.00490	0.00362	-0.00120	-0.00120	0.00057
1.44	-0.02265	-0.00980	0.00724	-0.00240	-0.00240	0.00114
2.16	-0.03398	-0.01470	0.01087	-0.00361	-0.00361	0.00171
2.88	-0.04531	-0.01960	0.01449	-0.00481	-0.00481	0.00228
3.61	-0.05679	-0.02457	0.01816	-0.00603	-0.00603	0.00286

Von Mises criteria in terms of internal forces was calculated using Eq. (4). Values of M_y and T_y calculated from Eq. (4) and (5) yield $M_y = 0.0520$ kNm and $T_y = 0.0374$ kNm. Ratio M/M_y and T/T_y of are shown at Table 4. Evaluation of Von Mises criteria is presented at Table 5 and Fig. 4. Based on Fig. 4, it can be seen that yielding affected by bending moment (M) is more dominant compared by torsion (T). Ratio analysis presented at Table 4 and 5 indicate that failure proportion affected by bending moment (M) is $\pm 90\%$ and $\pm 10\%$ affected by torsion (T).

Table 4. Ratio of bending moment and torsion with respect to capacity

Load (kN)	M/M_y			T/T_y		
	Point A	Point C	Point B	Point A	Point C	Point B
0.00	0.000	0.000	0.000	0.000	0.000	0.000
0.72	-0.218	-0.094	0.070	-0.032	-0.032	0.015
1.44	-0.436	-0.188	0.139	-0.064	-0.064	0.030
2.16	-0.653	-0.283	0.209	-0.096	-0.096	0.046
2.88	-0.871	-0.377	0.279	-0.128	-0.128	0.061
3.61	-1.092	-0.473	0.349	-0.161	-0.161	0.076

Table 5. Von Mises criteria evaluation

Load (kN)	$(M/M_y)^2+(T/T_y)^2$		
	Point A	Point C	Point B
0.00	0.000	0.000	0.000
0.72	0.048	0.010	0.005
1.44	0.194	0.040	0.020
2.16	0.436	0.089	0.046
2.88	0.776	0.159	0.081
3.61	1.219	0.249	0.128

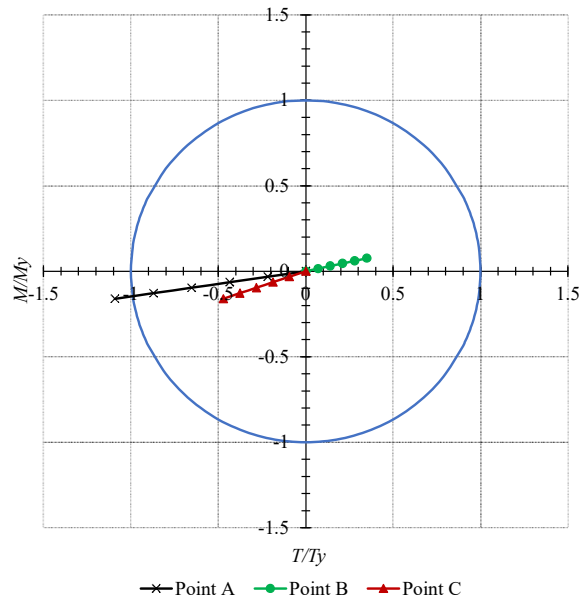


Fig. 4. Bending moment-torsion interaction.

Experimental results showed that plastic joint was initially formed at point A (left fixed constraint), followed at point C (right fixed constraint) and ultimately at point C (below the loading point). Model from DIANA FEA 10.3 predicted the exact same location compared with experimental result. Stress distribution at the section shows that the highest stress is located at the outer vertical of the beam section, this emphasizes that the failure is more affected by bending moment (M) rather than torsion (T). The detail of stress distribution is presented at Fig. 5.

Comparison of Model and Experimental Results of Elastoplastic Structure Loaded with Bending Moment and Torsion

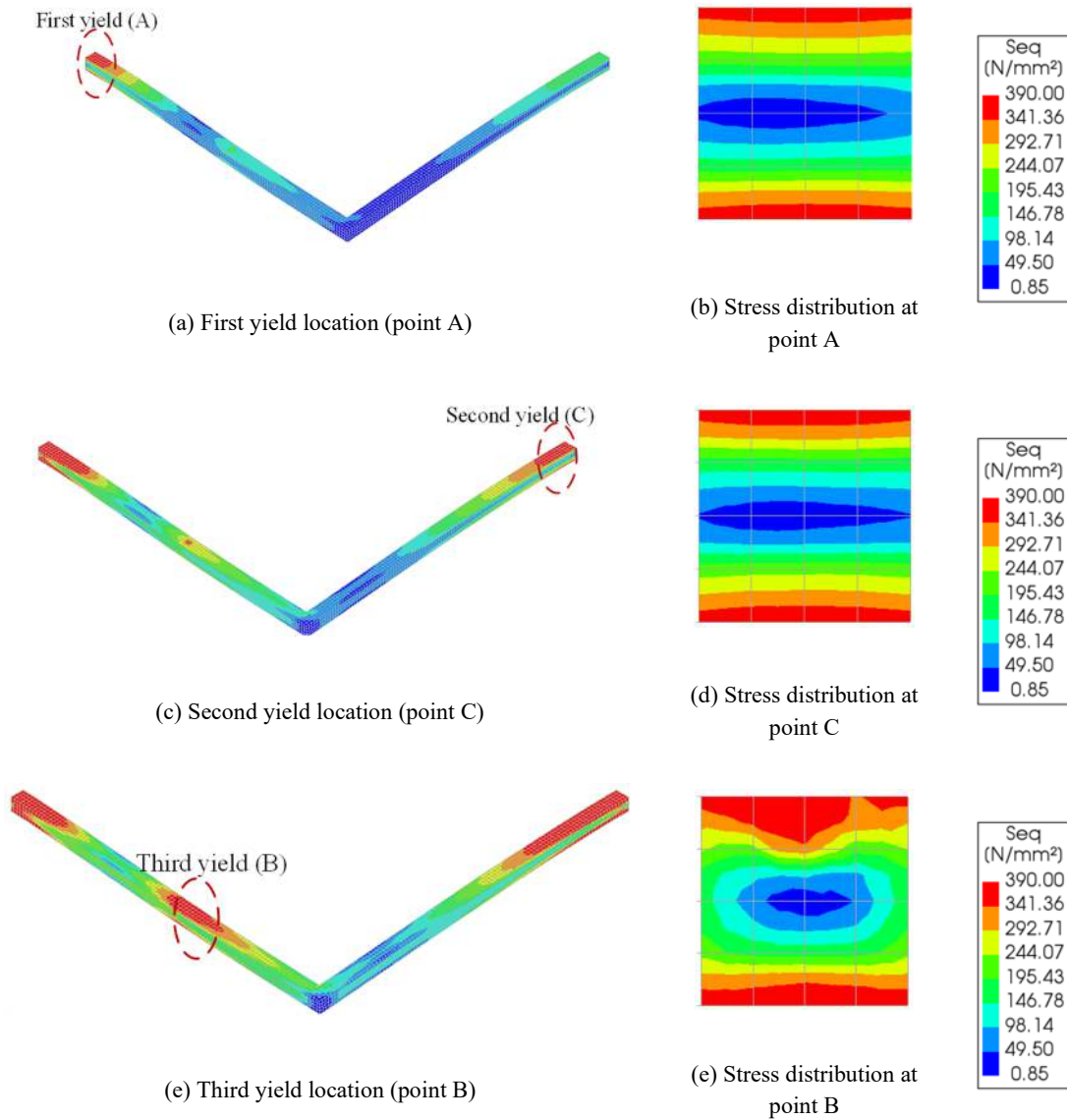


Fig. 5. Stress distribution of DIANA model.

5. Conclusion

Based on experimental result and numerical result from DIANA FEA 10.3 done for the 3D statically indetermined beam specimen tested with combination of bending moment (M) and torsion (T), it can be concluded that:

1. Experimental result and model from DIANA FEA 10.3 resulted in a representative output compared with previous study.
2. Bending moment (M) is the major factor in yielding process of the structure loaded with combination of bending moment (M) and torsion (T).
3. The elastoplastic behavior of 3D statically indetermined structure loaded with combination of bending moment (M) and torsion (T) can be observed.
4. Internal forces proportion in yielding process of 3D statically indetermined structure are $\pm 90\%$ affected by bending moment (M) and $\pm 10\%$ affected by torsion (T).

Acknowledgements

The authors gratefully acknowledge the financial support of the research funding by Directorate of Research, Technology and Community Service, Directorate General of Higher Education, Research and Technology, Ministry of Education, Culture, Research and Technology, the Republic of Indonesia through the decreed number 394-05/UN7.6.1/PP/2022.

References

- [1] Moestopo, M., (1996). On elastoplastic analysis of 1-D structural member subject to combined bending and torsion, *WIT Transactions on the Built Environment*, 17.
- [2] Refinda, (1997). *Modeling of Elastoplastic Behavior of Beam Sections under Combined Bending and Torsion*, Institut Teknologi Bandung, Bandung. (in Indonesia)
- [3] Pishro, A.A *et al.*, (2022). Structural Behavior of FRP-Retrofitted RC Beams under Combined Torsion and Bending, *Materials*, 15(9), 3213, <https://doi.org/10.3390/ma15093213>.
- [4] Konate, M., and Razzaq, Z. (2019). Yield limit interaction relations for biaxially loaded non-sway steel beam-columns with applied torsion. *Journal of Constructional Steel Research*, 156, 182-191.
- [5] Mohebkhah, A., and Azandariani, M. G. (2015). Lateral–torsional buckling of Delta hollow flange beams under moment gradient. *Thin-Walled Structures*, 86, 167-173.
- [6] Shokouhian, M., and Shi, Y. (2015). Flexural strength of hybrid steel I-beams based on slenderness. *Engineering Structures*, 93, 114-128.
- [7] Heyman, J., (1951). The Limit Design of Space Frames. *J Appl Mech*, 18,(2), 157–162, doi: 10.1115/1.4010270.
- [8] Chen, W. F., & Han, D. J. (2007). *Plasticity for structural engineers*. J. Ross publishing.
- [9] Dally, J.W., and Riley, W.F., (1978). *Experimental Stress Analysis International student edition*. McGraw-Hill.
- [10] Purwanto, (1997). *Experimental Study of the Elastoplastic Behavior of Steel Structures Subjected to Combined Bending and Torsion Loads* [Master’s Thesis, Institut Teknologi Bandung] (in Indonesia)