
Analysis of Students' Algebraic Reasoning Level in Learning Limit Function

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Abstrak. Penalaran aljabar penting bagi mahasiswa untuk dapat mengembangkan pemikiran secara sistematis dan analitis untuk mendapatkan suatu kesimpulan. Penelitian ini bertujuan untuk menganalisis tingkat penalaran aljabar mahasiswa pada pembelajaran limit fungsi. Metode yang digunakan yaitu metode kualitatif dengan teknik pengumpulan data menggunakan tes penalaran aljabar terhadap 23 mahasiswa semester satu program studi pendidikan matematika dan wawancara yang dilakukan dengan satu mahasiswa dari masing-masing tingkat tinggi, sedang, dan rendah. Teknik analisis data dalam penelitian ini terdiri dari pengumpulan data, reduksi data, penyajian data, dan kesimpulan. Dalam menyelesaikan tes penalaran aljabar, mahasiswa dengan tingkat tinggi dapat memenuhi empat dari lima indikator penalaran aljabar. Mahasiswa dengan tingkat penalaran aljabar sedang dapat memenuhi tiga dari lima indikator penalaran aljabar. Mahasiswa dengan tingkat penalaran aljabar rendah dapat memenuhi empat dari lima indikator penalaran aljabar. Mahasiswa dengan penalaran aljabar tinggi, sedang, dan rendah belum mampu dalam menemukan pola, membuat pola, dan menggeneralisasi pola.

Kata Kunci: penalaran aljabar, limit fungsi, generalisasi pola

Abstract. Algebraic reasoning is essential for students to develop thinking systematically and analytically to conclude. This study analyzes students' algebraic reasoning levels about learning limit functions. The method used is a qualitative method with data collection techniques using algebraic reasoning tests on 23 first-semester students of the mathematics education study program and interviews conducted with one student from each of the high, medium, and low levels. Data analysis techniques in this study consist of data collection, reduction, presentation, and conclusions. In completing the algebraic reasoning test, students with high levels can fulfill four of the five indicators of algebraic reasoning. Students with a moderate level of algebraic reasoning can fulfill three of the five indicators of algebraic reasoning. Students with a low level of algebraic reasoning can fulfill four of the five indicators of algebraic reasoning. Students with high, medium, and low algebraic reasoning cannot find patterns, create patterns, and generalize patterns.

Keywords: reasoning algebraic, function limit, generalized patterns

INTRODUCTION

Reasoning is the process of inferring based on existing evidence and assumptions. In mathematics, reasoning is deductive, which means interpreting from specific to general using axiomatic symbols (Shonia et al., 2020). One crucial component in improving the quality of education in Indonesia is reasoning ability (Shonia et al., 2020). Algebraic reasoning is more important than procedural skills, which tend to be mechanistic because mathematical material is more easily understood through reasoning (Hawes & Ansori, 2020).

Developing and improving algebraic reasoning is not an easy process. However, it can benefit students' future understanding of patterns and relationships in mathematics. The process of developing these abilities can be through mathematical problem-solving activities. Mathematics learning certainly supports this, where students are indirectly invited to solve problems gradually. The objectives of mathematics education include understanding mathematical concepts, explaining the relationship between concepts, and using concepts flexibly, accurately, efficiently, and precisely in problem-solving using reasoning (Panggabean et al., 2022).

One of the materials that can improve algebraic reasoning is the limit function material in the differential calculus course. This aligns with the statement that calculus is one of the courses in undergraduate programs in most science and engineering majors, including mathematics education (Sulastri, 2023). Furthermore, understanding function limits is students' initial stage of calculus learning.

Differential calculus is one of the compulsory courses given to students of the mathematics education study program at Sultan Agung Islamic University (UNISSULA). The learning material studied, especially the limit function material, is related to the ability to read, observe, and analyze images, graphs, patterns, etc. Suppose the concept of function and how to draw functions in graphs can be understood well. In that case, this understanding is beneficial in understanding the concept of limit and solving the limit using graph sketches (Sulastri, 2023).

The results of interviews with mathematics education students at UNISSULA show that, on average, students have not been able to understand the limited functions they have learned, including solving problems that lead to graph analysis and general conclusions such as finding patterns. Finding and understanding patterns in a mathematical problem and making generalizations using symbols is part of algebraic reasoning. (Indraswari & Zakiyah, 2020).

Algebraic reasoning is critical to learn because it is the basis of all mathematical thinking, including function limits that allow one to explore patterns in mathematics. However, the fact is that many students have not been able to understand algebra; one example is the course under study, differential calculus of limited function material. Students still have difficulty translating a problem into a mathematical model or function using the patterns that have been formed, so it impacts how they solve problems. This aligns with algebraic reasoning, an ability that focuses on the regularity of the problem-solving process. (Obara, 2019).

To support conceptual understanding of relationships in a formula, students must be able to explore relationships and make generalizations in algebraic reasoning. Algebraic reasoning indicators are benchmarks used to consider the achievement of algebraic reasoning. The indicators used in the study, according to Martin (2009), are as follows:

1. Using meaningful symbols. Identifying existing information, selecting variables building context in the form of expressions and equations, interpreting the form of expressions and equations, and transforming expressions to make exciting interpretations.
2. Connecting geometry to algebra. Solving problems by describing algebra-geometry and geometry-algebra situations and using these relationships.
3. We are connecting expressions and functions. Use multiple algebraic representations to understand functions and work with function notation.
4. Mind manipulation. We connect manipulations with arithmetic operations, anticipate the results of manipulations, select procedures appropriate to the situation, and perform mental arithmetic operations.

5. Reasonable solution. Exhibit solution steps as a logical understanding of relationships, finding patterns, defining rules, and generalizing patterns.

This research aims to analyze the level of students' algebraic reasoning in learning function limits. It is hoped that researchers can obtain accurate data to analyze students' algebraic reasoning as a reference for various parties in evaluating ongoing mathematics learning. Thus, the researcher chose "Analysis of Students' Algebraic Reasoning Level in Learning Limit Function."

RESEARCH METHODS

This research uses qualitative research methods. This study aims to analyze students' algebraic reasoning level in learning limit functions. The research was conducted in one of the private universities in Central Java in January 2024, and the subjects included as many as 23 first-semester students in the mathematics education study program. The data collection techniques used in this study were written tests given individually and online interviews through Google Meet. The data analysis process was carried out using an interactive model, according to Miles and Huberman. The data analysis process is carried out interactively and takes place continuously until completion, starting from data collection, data reduction, data visualization, and finally concluding/verification. The results of the algebraic reasoning test answers were confirmed using interviews. The algebraic reasoning test consists of one description question prepared by the researcher. Before being given to students, the question was validated by two supervisors considered valid and suitable for research. The algebraic reasoning test questions given to students are as follows:

A car travels fast with a velocity of 12.5 m/s when the time is close to five seconds. Determine the velocity of the car, which is five times the time minus half the time, and then square it.

- 1. What information do you get from the problem?*
- 2. Draw a graph that represents the information!*
- 3. What is the acceleration value (in $m/detik^2$) as t approaches two seconds, three seconds, and ten seconds?*
- 4. What is the value of acceleration at n seconds?*

The research subjects consisted of 3 students who were selected based on their level of algebraic reasoning ability. Students' algebraic reasoning is categorized as follows:

Table 1. Categorization of Student Test Results

Level	Interval	Number of Students
High	$rate > 73,01$	5
Medium	$64,81 \leq rate \leq 73,01$	14
Low	$rate < 64,81$	3

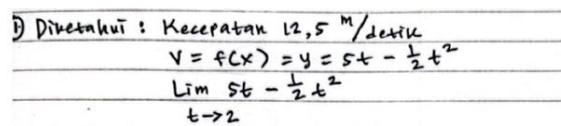
RESULT AND DISCUSSION

Based on the calculations that have been done, the average algebraic reasoning test score of 23 students is 68.91. The highest student score is 95, the lowest is 55, and the standard deviation is 8.20.

Based on the data, 3 students were selected to be interviewed: subject CY with high-level algebraic reasoning, subject NAF with medium-level algebraic reasoning, and subject SJ with low-level algebraic reasoning.

High Algebraic Reasoning

The answer to CY subject point A is as follows:



Diketahui: Kecepatan 12,5 m/detik
 $v = f(x) = 5t - \frac{1}{2}t^2$
 $\lim_{t \rightarrow 2} 5t - \frac{1}{2}t^2$

Figure 1. CY Answer Point A

Based on Figure 1, algebraic reasoning can be analyzed using indicators of meaningful symbol use. These indicators can be confirmed and explained in the interview results and descriptions below:

R : *What can you interpret from the expressions and equations based on the information you obtained?*

CY : *In the problem, a car travels at 12,5 m/s as it approaches five seconds. The car's velocity is five times minus half the time and then squared. [Intp]*

R : *How can you manipulate the expressions in the information to create exciting interpretations?*

CY : *By using symbols. Like velocity is symbolized by v , the manipulation form is as follows. Known: Velocity = 12,5 m/s, $v = 5t - \frac{1}{2}t^2$, and*

$\lim_{t \rightarrow 5} 5t - \frac{1}{2}t^2$. [Intro]

Based on the interview above, subject CY interpreted the information in the problem clearly without any errors. Then, subject CY was able to use symbols, such as velocity symbolized by v , mentioning velocity is equal to 12,5 m/s, $v = 5t - \frac{1}{2}t^2$, and $\lim_{t \rightarrow 5} 5t - \frac{1}{2}t^2$.

The answer to CY subject point B is as follows:

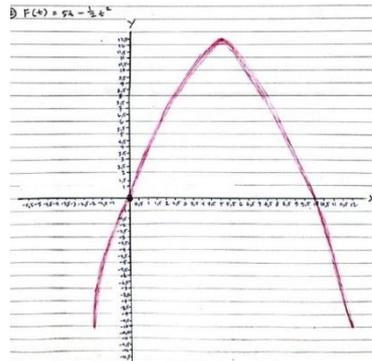


Figure 2. CY Answer Point B

Based on Figure 2. algebraic reasoning can be analyzed using indicators connecting algebra with geometry. These indicators can be confirmed and explained in the interview results and descriptions below:

R : *In what ways do you create graphic images?*

CY : ***By using GeoGebra. So, I entered the function first, and after I saw the graph image I determined the peak point. [Ag]***

Based on the interview results above, the CY subject made a graph with the help of GeoGebra. The way to make the graph is to enter the existing function first, then determine the peak point after the graph appears. Subject CY was able to make the graph correctly and accordingly.

In addition, from the answers in figure 2, algebraic reasoning can be analyzed using indicators of connecting expressions and functions. This indicator can be confirmed and explained in the interview results and description below:

R : *Does using algebraic representations to understand functions make it easier to solve the given problem?*

CY : ***It can make things easier. Because by using algebra like $5t - \frac{1}{2}t^2$ It can be made into a function that makes solving problems easier. [Ar]. However, we have to use the limit approach and the substitution method.***

Based on the results of the interview above, the CY subject uses algebra such as $5t - \frac{1}{2}t^2$ to understand the function so that it can make it easier to solve the given problem.

The answer to CY subject point C is as follows:

The figure shows handwritten mathematical work for seven limit problems, each with a corresponding numbered annotation:

- 1.** Calculating the when velocity approaching five seconds. (Problem #: $\lim_{t \rightarrow 5} 5t - \frac{1}{2}t^2$)
- 2.** Calculating the when velocity approaching two seconds. (Problem A): $\lim_{t \rightarrow 2} 5t - \frac{1}{2}t^2$)
- 3.** Calculating the when accelerated approaching two seconds. (Problem Δ): $\lim_{t \rightarrow 2} \frac{5t - \frac{1}{2}t^2}{t - 2}$)
- 4.** Calculating the when velocity approaching three seconds. (Problem b): $\lim_{t \rightarrow 3} 5t - \frac{1}{2}t^2$)
- 5.** Calculating the when accelerated approaching three seconds. (Problem Δ): $\lim_{t \rightarrow 3} \frac{5t - \frac{1}{2}t^2}{t - 3}$)
- 6.** Calculating the when velocity approaching ten seconds. (Problem c): $\lim_{t \rightarrow 10} 5t - \frac{1}{2}t^2$)
- 7.** Calculating the when accelerated approaching ten seconds. (Problem Δ): $\lim_{t \rightarrow 10} \frac{5t - \frac{1}{2}t^2}{t - 10}$)

Figure 3. CY Answer Point C

Based on Figure 3, algebraic reasoning can be analyzed using mind manipulation indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

R : *When answering this question, did you connect the manipulation with the arithmetic operation?*

CY : *Yes. In calculations, I use manipulation and relate it to arithmetic operations, such as subtraction, division, and multiplication. [Ao]*

Based on the interview results above, the CY subject can connect manipulation and arithmetic operations such as subtraction, division, and multiplication.

The answer to CY subject point D is as follows:

$$\begin{aligned} \textcircled{1} \quad a &= \lim_{t \rightarrow n} \frac{\Delta V}{\Delta t} \\ &= \lim_{t \rightarrow n} \frac{\sqrt{2} \ln 2 - \sqrt{1} \ln 2}{n_2 - n_1} \end{aligned}$$

Figure 4. CY Answer Point D

Based on Figure 4, algebraic reasoning can be analyzed using reasonable solution indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *How did you find the pattern from the solution you have done?*
 CY : *From the calculation results, I have not found a pattern.*
 R : *Explain what rule you used to find the pattern and how you defined the rule.*
 CY : *I can't explain the rule I used because I haven't been able to find a pattern in my answer.*
 R : *How do you generalize the pattern?*
 CY : *I do not know how to generalize the pattern.*

Based on the interview results above, the CY subject has not been able to find patterns in his calculations. In addition, the subject has not been able to determine the rules and does not know how to generalize patterns.

Medium Algebraic Reasoning

The answer to NAF subject point A is as follows:

$$\begin{aligned} 1.) \text{ Diketahui kecepatan mobil } &= 12,5 \text{ m/detik} \\ v &= at - \frac{1}{2}t^2 \\ v = f(x) = y &= at - \frac{1}{2}t^2 \end{aligned}$$

Figure 5. NAF Answer Point A

Based on Figure 5, algebraic reasoning can be analyzed using indicators of meaningful symbol use. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *What can you interpret from the expressions and equations based on the information you obtained?*
 NAF : *It is known that a car is traveling at 12,5 m/s when the velocity time is close to five seconds. In addition, it is known that the car's velocity is five times the time minus half times the time and then squared.*
 [Intp]

- R : *How can you manipulate the expression in the information to make an interesting interpretation?*
- NAF : *By using symbols. Time is symbolized by t , and velocity is symbolized by v , so the manipulation form is as follows: Kown: Velocity = 12,5 m/s and $v = 5t - \frac{1}{2}t^2$. [Intro]*

Based on the interview above, the NAF subject interpreted the information in the problem clearly without any errors. Then, the NAF subject used symbols in manipulating expressions, such as time is symbolized by t and velocity is symbolized by v , mentioning velocity is equal to 12,5 m/s and $v = 5t - \frac{1}{2}t^2$.

The answer to NAF subject point B is as follows:

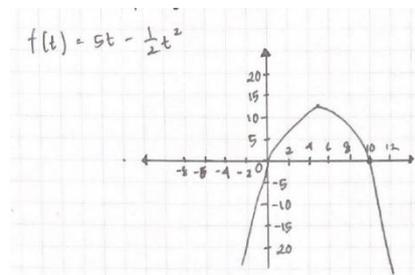


Figure 6. NAF Answer Point B

Based on Figure 6. algebraic reasoning can be analyzed using indicators connecting algebra with geometry. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *In what way did you draw the graph?*
- NAF : *By using GeoGebra. I entered the function into GeoGebra. After that, I saw the graph and found vertex 13.*

Based on the interview results above, the NAF subject made a graph with the help of GeoGebra. The way to make the graph is to enter the existing function first, then after the graph appears, determine the peak point of the graph. NAF subject said that the cusp of the graph is 13.

In addition, from the answers in Figure 6, algebraic reasoning can be analyzed using indicators of connecting expressions and functions. This indicator can be confirmed and explained in the interview results and description below:

- R : *Does using algebraic representation to understand functions make it easier to solve the given problem?*

NAF : *Yes. It can make it easier. Because algebraic representations such as $5t - \frac{1}{2}t^2$ If used as a function in the calculation, it can make it easier to solve the problem. [Ar]*

Based on the results of the interview above, NAF subjects use algebra such as $5t - \frac{1}{2}t^2$ to understand functions to make it easier to solve the problems given.

The answer to NAF subject point A is as follows:

1. Calculating the velocity when approaching two seconds.

2. Calculating the velocity when approaching three seconds.

3. Calculating the velocity when approaching ten seconds.

4. Calculating the when accelerated approaching two seconds.

5. Calculating the when accelerated approaching three seconds.

6. Calculating the when accelerated approaching ten seconds.

Figure 7. NAF Answer Point C

Based on Figure 7, algebraic reasoning can be analyzed using mind manipulation indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

R : *When answering this question, did you connect manipulation with arithmetic operations?*

NAF : *No, because I did not manipulate my calculation.*

Based on the interview results above, the NAF subject could not connect the manipulation with the arithmetic operation.

The answer to NAF subject point D is as follows:

$$a = \lim_{t \rightarrow n} \frac{\Delta V}{\Delta t}$$

$$= \lim_{t \rightarrow n} \left(\frac{v_2(n) - v_1(n)}{t - n} \right)$$

Figure 8. NAF Answer Point D

Based on Figure 8, algebraic reasoning can be analyzed using reasonable solution indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *How did you find the pattern from the solution you have done?*
 NAF : *I have not found a pattern from what I have done.*
 R : *Explain what rule you used to find the pattern and how you defined the rule.*
 NAF : *Because I have not been able to determine the pattern, I have not been able to define the rule.*
 R : *How do you generalize the pattern?*
 NAF : *I do not understand how to generalize patterns.*

Based on the interview results above, the NAF subject has not been able to find patterns from his calculations. NAF subject did not understand or generalize the pattern. The subject wrote the acceleration when approaching n in the answer because the formula applies in all velocity calculations. Based on this, the NAF subject could not find a reasonable solution. Because NAF subjects did not understand how to find and generalize patterns and define rules to recognize patterns.

Low Algebraic Reasoning

The answer to SJ subject point A is as follows:

$$\begin{aligned} \text{Kecepatan} &= 12,5 \text{ m/detik} \\ \text{Waktu} &= \text{Menentukan } t \text{ detik} \\ V &= 5t - \frac{1}{2}t^2 \\ V &= f(x) = y = 5t - t^2 \\ \lim_{t \rightarrow 5} 5t - \frac{1}{2}t^2 \end{aligned}$$

Figure 9. SJ Answer Point A

Based on Figure 9, algebraic reasoning can be analyzed using indicators of meaningful symbol use. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *What can you interpret from the expressions and equations according to the information you obtained?*
 SJ : *It is known that the velocity of the car is 12,5 m/s. It is also known that the car's velocity is five times the time minus half times the time and then squared. [Intp]*

Based on the interview above, subject SJ interpreted the information in the problem clearly without any errors. Based on this, subject SJ has not been able to fully use meaningful symbols because he has not been able to manipulate expressions to form exciting interpretations.

The answer to SJ subject point B is as follows:

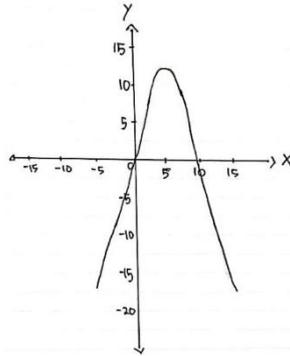


Figure 10. SJ Answer Point B

Based on Figure 10. Algebraic reasoning can be analyzed using indicators that connect algebra with geometry. These indicators can be confirmed and explained in the interview results and descriptions below:

R : *In what way did you create the graph image?*

SJ : *By using GeoGebra. Entering the function $5t - \frac{1}{2}t^2$ into GeoGebra, then drawing the graph, and then finding the cusp 12.*

Based on the interview results above, the subject SJ made a graph with the help of GeoGebra. The way to make the graph is to enter the existing function first, then determine the cusp of the graph after the graph appears. Subject SJ said that the cusp of the graph is 12.

In addition, from the answers in Figure 10, algebraic reasoning can be analyzed using indicators of connecting expressions and functions. This indicator can be confirmed and explained in the interview results and description below:

R : *Does using algebraic representation to understand functions make it easier to solve the given problem?*

SJ : *It is easier. Because algebra such as $5t - \frac{1}{2}t^2$, can be used to understand the function and of course facilitate the calculation [Ar].*

Based on the results of the interview above, subject SJ used algebra such as $5t - \frac{1}{2}t^2$ to understand the function so that it can make it easier to solve the given problem. Based on this, subject SJ has been unable to fully connect expressions and functions because he has not been able to use function notation properly.

The answer to SJ subject point C is as follows:

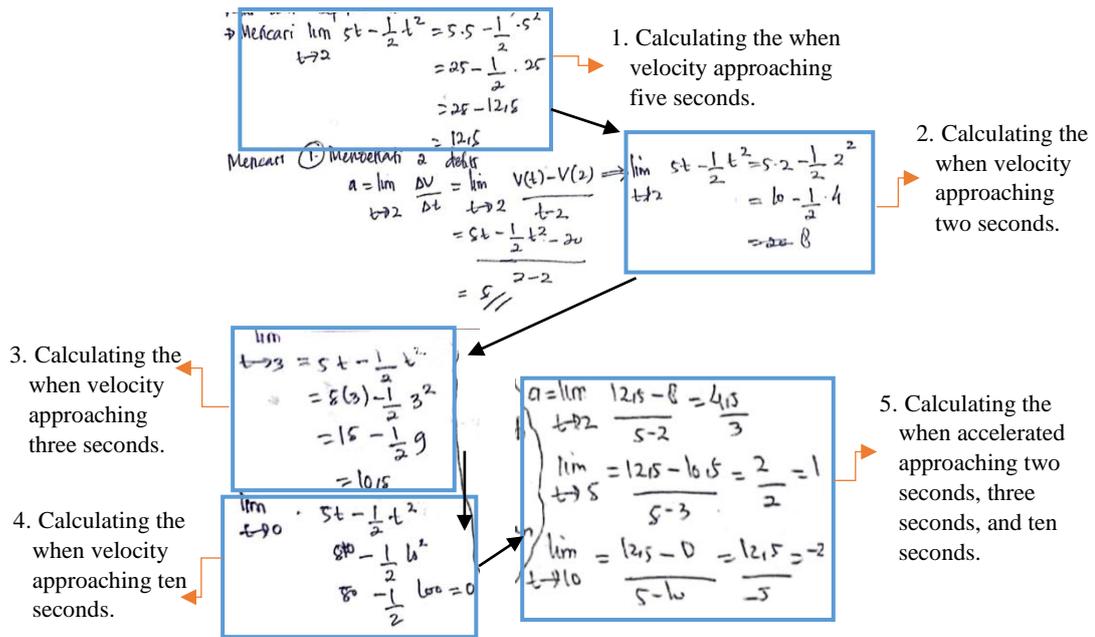


Figure 11. SJ Answer Point C

Based on Figure 11, algebraic reasoning can be analyzed using mind manipulation indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : When answering this question, did you connect the manipulation with the arithmetic operation?
- SJ : No, I did not because I didn't use manipulation in the calculation.
- R : How did you choose the appropriate solution procedure?
- SJ : I understood the problem first, then I used limit and substitution to solve the problem.
- R : Do you understand the steps that you have taken?
- SJ : I understand.
- R : Explain how you did the mental arithmetic.
- SJ : First, I used the substitution method in the function $5t - \frac{1}{2}t^2$ second, I calculated by using the limit and acceleration formula, for example $\lim_{t \rightarrow 2} \frac{v(t)-v(2)}{t-2}$.

Based on the interview results above, the SJ subject has been unable to connect manipulation. Then, the subject chooses the appropriate procedure by understanding the problem first. Using limits and substitution to solve the problem. Subject SJ can do mental arithmetic by substituting in the function $5t - \frac{1}{2}t^2$, and calculate by using the limit and acceleration formula, for example, $\lim_{t \rightarrow 2} \frac{v(t) - v(2)}{t - 2}$. Subject SJ has been unable to choose the appropriate solution procedure because the calculations still have errors.

The answer to SJ subject point D is as follows:

$$\begin{aligned}
 a &= \frac{\Delta v}{\Delta t} \\
 v_1 &= 12,5 \text{ m/s} \\
 v_2 &= 8 \times \left(5 - \frac{1}{2} \times 5^2\right) \\
 &= 8 \times \left(5 - \frac{25}{2}\right) \\
 &= 8 \times \left(\frac{10}{2} - \frac{25}{2}\right) \\
 &= 8 \times \left(\frac{10 - 25}{2}\right) \\
 &= 8 \times \left(\frac{-15}{2}\right) \\
 &= -60 \text{ m/s} \\
 \Delta v &= v_2 - v_1 \\
 &= -60 - 12,5 \\
 &= -72,5 \text{ m/s}
 \end{aligned}$$

Figure 12. SJ Answer Point D

Based on Figure 12, algebraic reasoning can be analyzed using reasonable solution indicators. These indicators can be confirmed and explained in the interview results and descriptions below:

- R : *How did you find the pattern from the solution you have done?*
 SJ : *I have not been able to find a pattern.*
 R : *Explain what rule you used to find the pattern and how you defined the rule.*
 SJ : *I do not understand how to find patterns, so I cannot define the rule yet.*
 R : *How do you generalize the pattern?*
 SJ : *I also do not understand how to generalize patterns.*

Based on the interview results, the SJ subject has not been able to find a pattern from his calculations. The subject does not understand generalizing patterns. The subject wrote the acceleration result when approaching n in the answer because it was according to the information in the problem. Based on this, the SJ subject has been unable to find a reasonable solution fully. Because the SJ subject is still

confused about how to find and generalize patterns and define rules to recognize patterns.

This study shows that students with high algebraic reasoning ability have not been able to fulfill the five indicators of algebraic reasoning in solving algebraic reasoning test questions on limit function material. Students with high, medium, and low algebraic reasoning abilities can describe and mention information when solving a problem. Students with high algebraic reasoning ability can use symbols, function notation, and limits entirely and correctly, such as v , t , and $v(t)$. Students with high reasoning ability can make and understand the function graph that has been made correctly and are fluent in explaining the calculation steps they have taken. Students with medium algebraic reasoning ability are fluent in calculating and explaining the steps of solving the problem but have not been able to make and understand the meaning of the graph they made. Students with low algebraic reasoning ability are less fluent in explaining and calculating the solution steps they have taken, have not been able to understand the graph, and have not been able to use function notation following the context of the problem.

Students with different levels of algebraic reasoning in solving problems number one and two-point A have in common, namely being able to use indicators of connecting expressions and functions. This is because questions number one and two, point A presented, are easy to do, one of which is by making algebraic representations. However, in solving problem number two, point B, students experienced significant differences at each level.

Based on the five indicators used by researchers in this study, students with high, medium, and low algebraic reasoning in solving algebraic reasoning problems can interpret information and make algebraic representations that exist in solving algebraic reasoning tests. However, students with high, medium, and low levels have been unable to find the final solution, namely finding patterns, making patterns, and generalizing patterns.

In the indicator of using meaningful symbols related to interpreting information to make exciting interpretations, students with high, medium, and low

algebraic reasoning have almost the same ability related to this indicator. Students with high, medium, and low levels can interpret correctly.

Indicators of connecting algebra with geometry related to graphs: high-ability students make graphs correctly, altogether, and accordingly and can understand the meaning of the graphs made. Students with moderate ability are pretty capable of making graphs but have not been able to explain the graphs they have made. Students with low ability have not been able to make correct graphs. This can be related to student curiosity in solving a problem or problem. High-ability students tend to be more curious.

Meanwhile, students with moderate ability are pretty curious, and low-ability students only know but do not understand what they have done. Some curious people will look for detailed information about everything that is asked of them. They will try to answer every question that comes to mind through curiosity, providing a new experience and information (Zetriuslita, 2016).

Students with high ability tend to be more critical because in solving problems related to graphs, high-ability students will first analyze, such as enlarging and reducing the graph until they find the peak or intersection point. Medium-ability students do not analyze the graph obtained; low-ability students only know enough without analyzing it. This aligns with research conducted by Alvionita et al. (2019); students are not confident in learning math and have no desire to solve problems. As a result, low-ability students tend to lack curiosity.

The indicator of connecting expressions and functions related to function notation, low ability students still do not understand the function notation used in solving the problem. However, low-ability students can already use algebraic representations to understand functions. High- and medium-ability students can make correct function notations and use algebraic representation.

Students with high ability can connect manipulation with arithmetic operations, such as using manipulation with commutative properties in solving number 1 point A. Meanwhile, students with medium and low ability cannot connect manipulation with arithmetic operations. Students with high ability perform calculations in more detail and according to the steps. This is because high-

ability students have a great curiosity in solving a problem. A sense of curiosity will motivate students to find answers to the problems given (Zetriuslita, 2016).

Students with low ability to manipulate minds, especially in completing calculations, tend to be more general and do not use detailed steps. So, in this case, there is an indirect sense of curiosity because students only complete as much as possible. This is also influenced by a lack of confidence in solving problems and not being sure of getting good results (Alvionita et al., 2019).

Indicators of reasonable solutions related to finding patterns, making patterns, and generalizing patterns. High-, medium-, and low-ability students have the same ability but cannot solve problems related to these indicators yet. For high, medium, and low-ability students, finding patterns to make patterns is new, so when solving the problems given, all students with different levels still feel unfamiliar.

Students' algebraic reasoning can be improved through curiosity in solving all the problems given. In the learning process, an attitude of curiosity will encourage students to study, explore, and explore unknown information (Rezi et al., 2024). In addition, in improving algebraic reasoning, it is necessary to provide various types according to the level of instruction the teacher gives to students to solve the problems or problems given (Basir et al., 2022).

CONCLUSION

Based on the results of research and discussion, it can be concluded that students with high algebraic reasoning ability have a greater level of curiosity in solving the problems given, can analyze, and are critical in explaining the completion steps. Students with moderate algebraic reasoning ability have ordinary curiosity, enough to analyze, and smooth enough to explain the completion steps. Students with low algebraic reasoning ability have a sense of insecurity that creates a sense of ignorance in solving the problems given, and this makes students with low algebraic reasoning not understand what they have done and feel confused when asked to explain again. Of the three abilities, they have in common that they have not been able to generalize patterns or have not been able to meet the indicators of a reasonable solution.

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